

# A FOOTNOTE TO EINSTEIN 1907

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## Abstract

An add-on to Einstein's equivalence principle of 1907, specifically to its third point, is offered. The three points read: (1) *gravitational redshift*, (2) *gravitational light-ray bending*, and (3) *local reduction of the speed of light  $c$* . The third point can be replaced by a more general effect that explains it: (3') *gravitational light-ray tilting*. The consequence is "global constancy of the speed of light  $c$ ." This retrieved implication of the equivalence principle is bound to carry over to general relativity. A new "analogical" subfield of digital numerical simulations is called for.

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## Introduction

The most daring because at first sight most absurd paper in the history of physics is Einstein's 1907 intuitive proposal of the "equivalence principle" [Einstein, 1907] The name means: hypothetical equivalence between gravitation and ordinary kinematic acceleration. The principle needs refinement, as Rindler [1968] and Bell [1987] showed (height-dependent differential rather than uniform acceleration). However, even in its original form the equivalence principle proves more powerful than hitherto thought. Einstein's *third* implication, after "gravitational redshift" and "gravitational light bending," reads: "gravitational reduction of the speed of light  $c$ " (cf. second-but-last page of [Einstein, 1907]). The latter phenomenon exists but turns out to be only a *projection effect*. If so, the axiom of a globally constant  $c$  – formally implicit in the purely special-relativistic scenario of the equivalence principle – is retrieved as is logically required. The new more complete understanding of the equivalence principle is sketched in the following.

### **First Implication of the Einstein Rocketship: Gravitational Redshift**

The first gravitational result of Einstein's is implicitly known to every user of a GPS based navigation system: all clocks down here on earth tick slower than those in the satellites overhead. Einstein foresaw this fact in his analogical model of a *constantly accelerating long rocketship in outer space* of December 1907 [Einstein, 1907]. He argued as follows: since the speed of light inside the ignited long rocketship is not infinite but only finite, a light ray emitted from the bottom reaches the tip only when the rocketship including the tip has picked up a certain amount of speed. Therefore the ascending light wave possesses a constant *Doppler-reduced frequency* relative to its point of arrival at the tip. This “gravitational redshift” is an epoch-making finding based on analogical thinking.

### **Second Implication of the Einstein Rocketship: Gravitational Light-ray Bending**

A horizontally emitted light ray – say: in the middle part of the accelerating long rocketship – curves gently downwards toward the side in a parabolic shape. This is because the rocketship continues picking up speed itself while the light is traveling sideways. The new “gravitational light bending” is only a second-order effect. It likewise represents standard knowledge in the “flat metric” of the equivalence principle.

### **Traditional Third Implication of the Einstein Rocketship: Gravitational Reduction of $c$**

From the point of view of the tip, a light ray emitted horizontally at the bottom appears not only redshifted (first prediction) and gently downward-bending (second prediction) but also *slowed* by the redshift factor (third prediction). The visible-from-above reduction in the speed of light  $c$  amounted to a major drawback for Einstein since the rocketship is governed by special relativity with its globally constant  $c$ . Einstein at this point fell silent on the topic of gravitation for 3 ½ years up until June 1911, a fact that is so far unexplained [Pais, 2005].

### **Completed Third Implication of the Einstein Rocketship: Gravitational Light-ray Tilting**

This new first-order effect is on a par with gravitational redshift itself. It is proportional to the latter and likewise locally masked. It goes like this: Relative to the tip, a light ray hugging the horizontal bottom is – in addition to being redshifted and gently curved and optically slowed when viewed from above – *tilted* relative to the tip and therefore *elongated* relative to its vertical projection. As a consequence, the speed of light  $c$  is a global constant again.

This effect comes as a surprise after more than a century. To appreciate the new proposal, it is necessary to have a closer look at a light ray grazing the horizontal bottom of the rocketship (assuming for convenience a tiny vertical distance to allow for some bending, and a bit of glitter dust suspended in the air to make the passing light pulse visible from above). Then the light pulse down there looks *reduced* in its speed when watched from above. This fact Einstein convincingly showed on the second-but-last page of [Einstein, 1907]. This *third* discovery is not the end of the story, however. The optically reduced speed of light reflects a *further*, apparently not previously described, fact: The locally horizontal light ray downstairs is everywhere locally *tilted* relative to the tip! This new fact based on analogical visualization makes the horizontal ray *longer* in exact compensation for the optical shrinking.

While the horizontal light ray downstairs advances along the floor, the floor is constantly *receding* from the tip generating the *gravitational redshift*. And generating the *gravitational tilt*. The constant *tilt* parallels the constant *recession*. The tilt is zero at zero height difference and increases at larger vertical distances. It implies a visual shortening (like the shortened shadow of a non-transversally held pencil in the midday sun) which explains the optically reduced speed of light  $c$  seen by Einstein [1907].

The tilt-without-tilt parallels the recession-without-recession seen by Einstein. It is just as real: when the *next* horizontal point on the bottom is reached by the laterally progressing light ray, the tip has already picked up a somewhat higher vertical speed. The *tilting* and hence *light-ray elongating effect* got overlooked so far. Both effects have a subtly fractal character.

## Discussion

A footnote to Einstein 1907 was described: *gravitational light-ray tilting relative to above*. The new effect *complements* the gravitational light-ray stretching relative to above of 1907 (#1). It at the same time *completes* the third implication of the Einstein rocketship, the visible reduction of the speed of light  $c$  (#3). The latter gets both explained and compensated-for (#3'). The new spatial elongation relative to the tip *parallels* the temporal elongation relative to the tip of the *gravitational redshift*. Like the latter it is almost impossible to visualize. Computer-based visualization techniques will prove crucial in the end. Presently it appears that the new horizontal corollary to the vertical gravitational light ray elongation relative to the tip seen by Einstein is as real as the latter. If so, a new stepping stone has been reached.

The consequence is *global constancy of the speed of light  $c$*  in the equivalence principle and by implication beyond. This proposal upsets more than a century old views. Therefore the reader will no doubt object at this

point that “results derived by analogical thinking are not scientific.” Einstein later got healed from presenting results based on analog-type spatial reasoning by using only equations in his future deductions. Is the above old-style result not intrinsically flawed?

This is quite possible. The analogical visualization employed is amazingly difficult to uphold. A full description of the simple Einstein rocketship – even under the idealizing assumption made of vertically uniform acceleration – remains elusive. *If* the above result of global constancy of  $c$  in the equivalence principle is robust, the consequences are far-reaching (for example, for cosmology). Search for a counterproof to the above “analogical conjecture” of *c-global in the equivalence principle of special relativity* thereby acquires some priority and even juiciness.

To conclude, a new angle was thrown on the “exact visual imagery” invented by a still “analogically minded” Einstein in his seminal paper on gravitation of 1907. A new branch of computer animation reproducing the above distance- and time-dependent distortion effects “à la Walt Disney” (who loved smooth distortion effects in his hand-drawn movies) is encouraged owing to the importance of the above spatio-temporal finding provided it holds water. The “analogical legacy of Einstein” waits to be harvested. One first fruit was tentatively offered: “global- $c$ , regained.”

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